BIOLOGICAL CONTROL BASED ON THE SYNCHRONIZATION OF LOTKA-VOLTERRA SYSTEMS WITH FOUR COMPETITIVE SPECIES

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Abstract. In this work we present the synchronization of two Lotka-Volterra with four competitive species in order to formulate the biological control with two preys, two predators. Our results show that the transient until synchronization depends on the initial conditions of two systems and on the control strength. For practical reasons we need a single controller in order to achieve the synchronization; for these systems the synchronization is about four times faster when we use all the controllers than when we use a single controller. In addition the only way we can use a single controller is if we apply it in the first or in the third equation. If we must interfere on the second population, we must use two controllers. We suggest that we can control the pests by synchronizing the pest population with the population of the parasitoid by varying the initial conditions and the control strength.

Key words: predator-prey system, pest control, synchronization.

INTRODUCTION

The pest control is of great interest in agriculture domain because pests have been the major factor that reduces the agricultural production in the world. Different methods are being used in the process of pest management, for instance, chemical pesticides, biological pesticides, computers, atomic energy [8, 13], etc. Of all the methods, chemical pesticides seem to be a convenient and efficient one, because they can quickly kill a significant portion of the pest population. But synthetic chemical pesticides that have been introduced and that are being widely used on agricultural crops, in order to control the agricultural pests, represent a significant food safety risk [2, 14]. Organic agriculture imposes biological control which uses living organisms to suppress pest populations.

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Periodic releases of the parasitoid *Encarsia formosa* were used to control greenhouse whitefly, and the predaceous mite *Phytoseiulus persimilis* was used for the control of the two-spotted spider mite. There is a vast amount of literature on the applications of microbial disease to suppress pests. At the same time, there are only a few works on the mathematical models of the dynamics of microbial diseases in pest management. Very recently Yuanshun Tan and Chen [17] used a simple mathematical model for pest control by impulsively releasing infected pests.

There are different approaches in regard to the possibility of modeling agricultural systems. One of the famous examples of a simple model is the logistic map which can model the complex dynamics of some real population systems. The Lotka-Volterra model is widely used to study the dynamics of interacting species. In this case, the prey-predator or host parasitoid models ignore many important factors such as interactions between other species of the same ecosystem, interactions with the environment, etc. Arneodo *et al.* [1] have demonstrated that one can obtain chaotic behaviour for three species. In a 1988 paper Samardzija and Greller [15] propose a two-predator, one prey generalization of the Lotka-Volterra problem into three dimensions. The synchronization of the trajectories of the two attractors of this modified, three-dimensional Lotka-Volterra equation, was performed by John Costello [3] using the Kapitaniak method. In addition Deng and Loladze [4] argued that the classical predator-prey models, such as Lotka-Volterra, track the abundance of prey, but ignore its quality. Therefore, the authors showed that each organism is a mixture of multiple chemical elements and that the ratios of these elements can vary within and among species (these ratios represent prey quality). When these ratios vary, as they frequently do in nature, seemingly paradoxical results can arise such as the extinction of a predator that has an abundant and accessible prey. The authors suggested that, when competing predators differ in their sensitivity to prey quality, then all species can coexist via chaotic fluctuations.

On the basis of the Lotka-Volterra model, the pest control was formulated by Rafikov [13, 14] analyzing the relations between two soybean caterpillars (*Rachiplusia nu* and *Pseudoplusia includens*) supposed to be parasitoids (considering that there are soybean plants in abundance). The main scope of the authors was to obtain a pest control strategy through the introduction of natural enemies. The control needs to move the system to the steady state; this means that the pest density is stabilized without causing economic damages, and that the natural enemies' population is stabilized at a level that can control the pests. The numerical simulations of the authors based on one prey - one predator Lotka-Volterra model showed that control strategy can maintain the pest population below injury level for a long time. For a Lotka-Volterra model with three species, two preys - one predator, the analytical and numerical studies revealed that control strategies could not control the pest population below the economic injury level. The pest control problem was resolved for a two preys - two predators Lotka-

Volterra model. In this case, the two caterpillars were considered as different species and one new parasitoid was introduced into the system, and the ecosystem was modeled by four differential equations. Over the past decade, there has been considerable progress in the generalization of the concept of synchronization to include the case of coupled chaotic oscillators especially for biological systems. When the complete synchronization is achieved, the states of both systems become practically identical, while their dynamics in time remains chaotic [5, 6, 10, 11, 12]. Many examples of biological synchronization have been documented in the literature, but currently theoretical understanding of the phenomena lags behind experimental studies.

Sprott [16] found a high-dimensional Lotka-Volterra model (many-species) that exhibits spatiotemporal chaos. The authors considered the simplest form of such a system in which *N* species with population x_i (for $i = 1$ to N) compete for a finite set of resources according to:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \sum_{j=1}^{N} a_{ij} x_j\right) \tag{1}$$

Here r_i represents the growth rate of species i and a_{ij} represents the extent to which species j competes for resources used by species i. For the single-species case, i.e. $N = 1$, the equation (1) is reduced to the logistic equation. In the two-species case, i.e. $N = 2$, there is a coexisting equilibrium. For three species chaotic solutions of equation (1) are possible.

Vano [18] studied the occurrence of chaos in basic Lotka-Volterra models of four competing species. They found chaos in a four-species competitive Lotka-Volterra model for values such as:

$$r_i = \begin{bmatrix} 1 \\ 0.72 \\ 0.53 \\ 1.27 \end{bmatrix} \qquad a_{il} = \begin{bmatrix} 1 & 1.09 & 1.52 & 0 \\ 0 & 1 & 0.44 & 1.36 \\ 2.33 & 0 & 1 & 0.47 \\ 1.21 & 0.51 & 0.35 & 1 \end{bmatrix} \tag{2}$$

Figure 1 shows the attractor projected onto x_1, x_3, x_4 space for the chaotic system (1) with values from (2).

To synchronize two Lotka-Volterra systems with four competitive species, we used a simple method for chaos synchronization proposed in [7, 9].

If the chaotic system (master) is:

$$\dot{x} = f(x) \tag{3}$$

where

$$x = (x_1, x_2, ...x_n) \in R^n \;\; f(x) = (f_1(x), f_2(x).....f_n(x)) : R^n \to R^n \tag{4}$$

then the slave system is of the form:

$$\dot{y} = f(y) + z(y - x) \tag{5}$$

where the function z can be chosen as:

$$\dot{z} = -(y - x)^2 \tag{6}$$

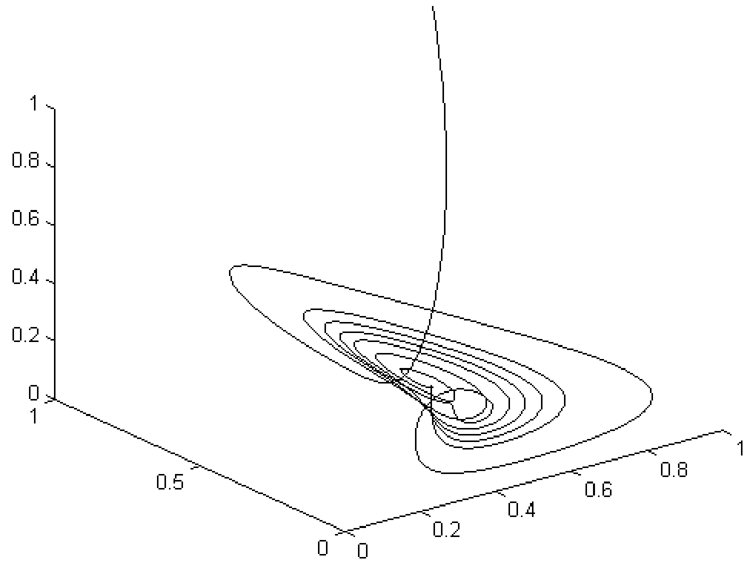

Fig. 1. Phase portrait of (x_1, x_3, x_4) for Lotka-Volterra generalized system with four competitive species [$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$].

RESULTS

CASE 1, ALL CONTROLLERS

From (1) and (2) the master system is:

$$\dot{x}_1 = x_1(1 - x_1 - 1.09x_2 - 1.52x_3)$$

$$\dot{x}_2 = 0.72x_2(1 - x_2 - 0.44x_3 - 1.36x_4)$$

$$\dot{x}_3 = 1.53x_3(1 - 2.33x_1 - x_3 - 0.47x_4)$$

$$\dot{x}_4 = 1.27x_4(1 - 1.21x_1 - 0.51x_2 - 0.35x_3 - x_4) \tag{7}$$

The slave system with four controllers becomes:

$$\dot{y}_1 = y_1(1 - y_1 - 1.09y_2 - 1.52y_3) + z_1(y_1 - x_1)$$

$$\dot{y}_2 = 0.72y_2(1 - y_2 - 0.44y_3 - 1.36y_4) + z_2(y_2 - x_2)$$

$$\dot{y}_3 = 1.53y_3(1 - 2.33y_1 - y_3 - 0.47y_4) + z_3(y_3 - x_3)$$

$$\dot{y}_4 = 1.27y_4(1 - 1.21y_1 - 0.51y_2 - 0.35y_3 - y_4) + z_4(y_4 - x_4) \quad (8)$$

and the control strength is of the form:

$$\dot{z}_1 = -(y_1 - x_1)^2$$

$$\dot{z}_2 = -(y_2 - x_2)^2$$

$$\dot{z}_3 = -(y_3 - x_3)^2 \quad (9)$$

$$\dot{z}_4 = -(y_4 - x_4)^2 \quad (9)$$

The synchronization is very fast as Figs. 2–5 show.

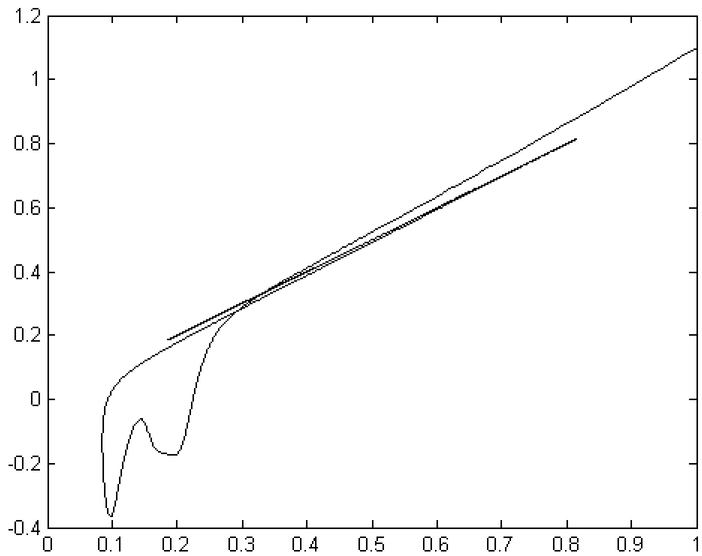

Fig. 2. Phase portrait of (x_4, y_4) for Lotka-Volterra generalized system with four competitive species $[x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1, y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1.1, z_1(0) = z_2(0) = z_3(0) = z_4(0) = 1]$.

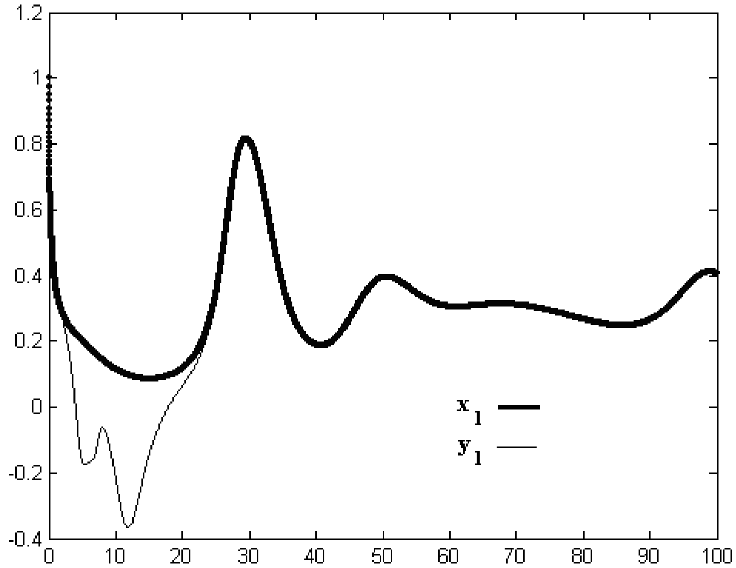

Fig. 3. $x_1(t)$, $y_1(t)$, [$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$, $y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1.1$, $z_1(0) = z_2(0) = z_3(0) = z_4(0) = 1$].

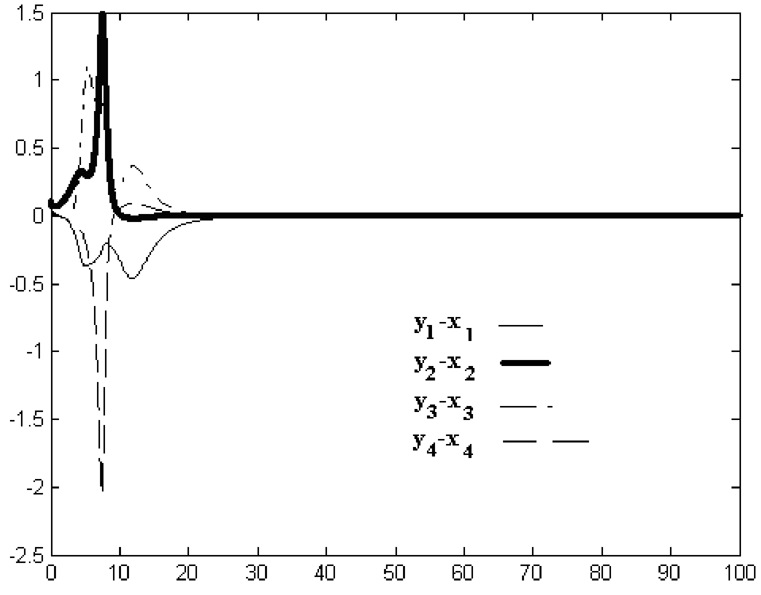

Fig. 4. Synchronization errors between master and slave systems [$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$, $y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1.1$, $z_1(0) = z_2(0) = z_3(0) = z_4(0) = 1$].

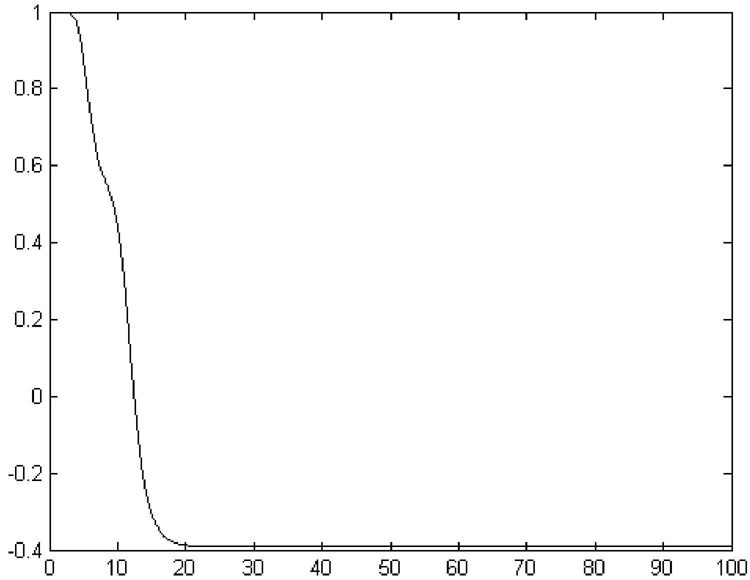

Fig. 5. The control strength z_1 [$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$, $y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1.1$, $z_1(0) = z_2(0) = z_3(0) = z_4(0) = 1$].

CASE 2, ONE CONTROLLER

Debin Huang [9], by testing the chaotic systems including the Lorenz system, Rossler system, Chua's circuit, and the Sprott's collection of the simplest chaotic flows, found that coupling only one variable is sufficient to achieve identical synchronization of a three-dimensional system.

For practical applications we need a single controller. In [12] we synchronized the Lorenz system using a single controller in any of the equations. In [6] we were able to synchronize two electric circuits experimentally. Now we are studying the use of a single controller in order to synchronize two Lotka-Volterra systems with 4 species so that we can later perform the synchronization experimentally. This theoretical research on the ways that the synchronization can be practically used is what this paper brings, in comparison with other authors who researched chaotic behavior of Lotka-Volterra system [13, 14].

For system (1), the only way to achieve synchronization using a single controller is to apply it in the first or in the third equation of the system. For example, if the controller is in first equation, the synchronization is given in Fig. 6 but it is slower than the first case. If the controller is used in the second or in the fourth equation, species 1 collapses, as Fig. 7 shows.

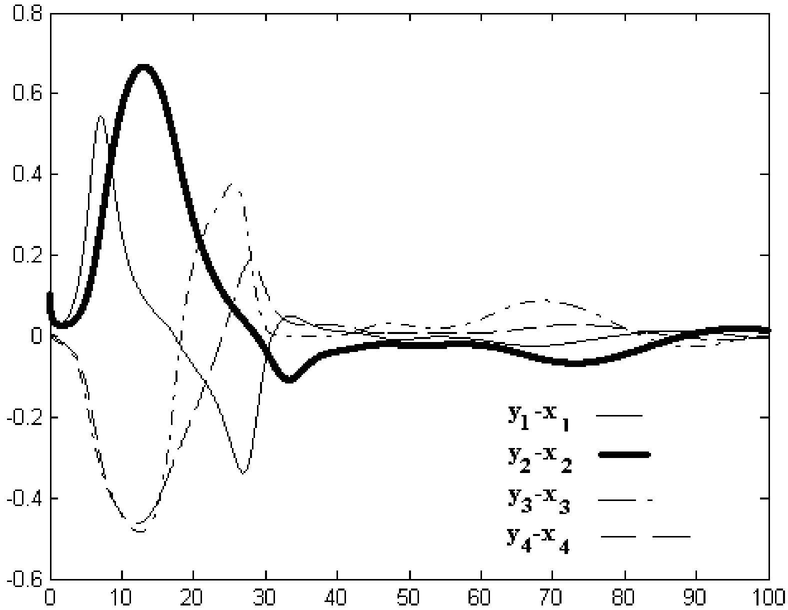

Fig. 6. Synchronization errors between master and slave systems for one controller
[$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$, $y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1.1$, $z_1(0) = 1$].

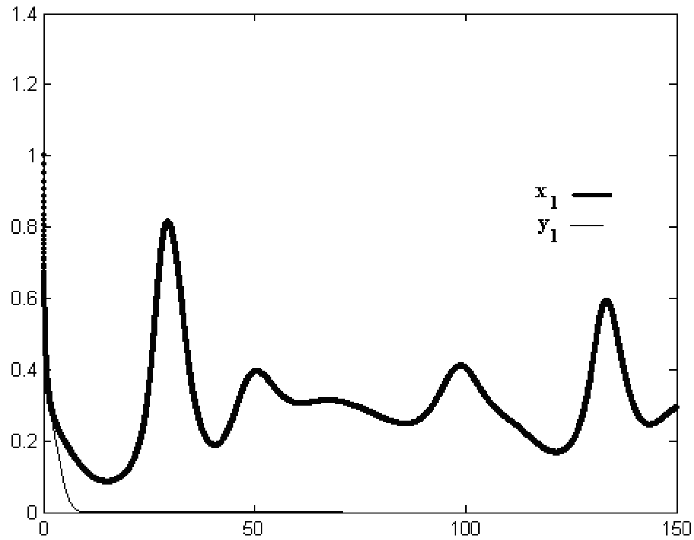

Fig. 7. $x_1(t)$, $y_1(t)$, [$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$, $y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1.1$, $z_2(0) = 1$].

Therefore, in order to control all the species using a single controller, this controller must be used in equation (1) or in equation (3) of the system (7).

CASE 3, TWO CONTROLLERS

If we have to interfere on the second population, we must use two controllers. For two controllers in the first and the second equation, the synchronization errors are given in Fig. 8.

For these two controllers the synchronization is achieved later than in the earlier cases.

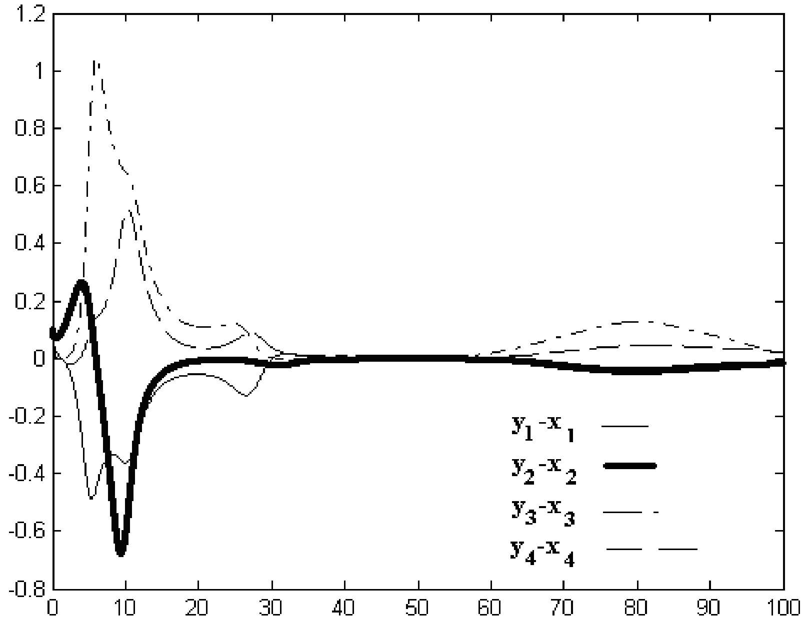

Fig. 8. Synchronization errors between master and slave systems for two controllers [$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 1$, $y_1(0) = y_2(0) = y_3(0) = y_4(0) = 1.1$, $z_1(0) = z_2(0) = 1$].

CONCLUSIONS

In this work we present the synchronization of two Lotka-Volterra with four competitive species in order to formulate the biological control with two preys - two predators. Our results show that the transient time until synchronization depends on initial conditions of two systems and on the controller's number. For practical reasons we need a single controller in order to achieve the synchronization; for these systems the syncronization is about four times faster when we use all the controllers than when we use a single controller. In addition, the only way we can use a single controller is if we apply it in the first or in the third equation. If we have to interfere on the second population, we must use two controllers. For two controllers the synchronization is achieved later than in the earlier cases. The control method described in this paper is very easy and might be useful in the case of other chaotic systems. We suggest that we can control the pests by synchronizing the pest population with the population of the parasitoid by varying the initial conditions and the control number.

REFERENCES

1. ARNEODO, A., P. COULLET, C. TRESSER, Occurrence of strange attractors in three-dimensional Volterra equation, *Physics Letters A*, 1980, **79**, 259–263.
2. BHATTACHARYYA, S., D.K. BHATTACHARYA, Pest control through viral disease: Mathematical modeling and analysis, *Journal of Theoretical Biology*, 2006, **238**, 177–197.
3. COSTELLO, J.S., Synchronization of chaos in a generalized Lotka-Volterra attractor, *The Nonlinear Journal*, 1999, **1**, 11–17.
4. DENG, B., I. LOLADZE, Competitive coexistence in stoichiometric chaos, Chaos, 2007, **17**, 1–14.
5. GROSU, I., Robust synchronization, *Phys. Rev.*, 1997, **56**, 3709–3712.
6. GROSU, I., E. PADMANABAN, P.K. ROY, S.K. DANA, Designing coupling for synchronization and amplification of chaos, *Physics Letters A*, 2008, **100**, 234102–234104.
7. GUO, W., S. CHEN, H. ZHOU, A simple adaptive-feedback controller for chaos synchronization, *Chaos, Solitons and Fractals*, 2009, **39**, 316–321.
8. HASHEMI, S.M., S.M. HOSSEINI, C.A. DAMALAS, Farmers' competence and training needs on pest management practices: Participation in extension workshops, *Crop. Protection,* 2009, **28**, 934–939.
9. HUANG, D., Simple adaptive-feedback controller for identical chaos synchronization, *Phys. Rev. E*, 2005, **71**(2B), 0372031–0372034.
10. LERESCU, A.I., N. CONSTANDACHE, S. OANCEA, I. GROSU., Collection of master-slave synchronized chaotic systems, *Chaos, Solitons and Fractals*, 2004, **22**(3), 599–604.
11. LERESCU, A.I., S. OANCEA, I. GROSU, Collection of mutually synchronized chaotic systems, *Physics Letters A*, 2006, **352**, 222–228.
12. OANCEA, S., F. GROSU, A. LAZAR, IOAN GROSU, Master-slave synchronization of Lorenz systems using a single controller, *Chaos, Solitons and Fractals*, 2009, **41**, 2575–2580.
13. RAFIKOV, M., J.M. BALTHAZAR, H.F. VON BREMEN, Management of complex systems, Modeling the biological pest control, *Biophysical Reviews and Letters*, 2008, **3**(1, 2), 241–256.
14. RAFIKOV, M., J.M. BALTHAZAR, H.F. VON BREMEN, Mathematical modeling and control of population systems: Applications in biological pest control, *Applied Mathematics and Computation*, 2008, **200**, 557–573.
15. SAMARDZIJA, N., L.D. GRELLER, Explosive route to chaos through a fractal torus in a generalized Lotka-Volterra model, *Bulletin of Mathematical Biology*, 1988, **50** (5), 465–491.
16. SPROTT, J.C., J.C. WILDENBERG, Y. AZIZI, A simple spatiotemporal chaotic Lotka-Volterra model, *Chaos, Solitons and Fractals*, 2005, **26**, 1035–1043.
17. TAN, Y., L. CHEN, Modeling approach for biological control of insect pest by releasing infected pests, *Chaos, Solitons and Fractals*, 2009, **39**, 304–315.
18. VANO, J.A., J.C. WILDENBERG, M.B. ANDERSON, J.K. NOEL, J.C. SPROTT, Chaos in low-dimensional Lotka-Volterra models of competition, *Nonlinearity*, 2006, **19**, 2391–2404.