# SIMPLIFIED ANALYTICAL DOSIMETRY FOR OPHTHALMIC BRACHYTHERAPY: A PRACTICAL ALTERNATIVE TO MONTE CARLO SIMULATIONS

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*Abstract*. The aim of this study was to develop and evaluate a simplified dosimetric methodology for ophthalmic brachytherapy, providing a practical alternative to Monte Carlo (MC) simulations. A mathematical eye model incorporating major ocular structures was employed. Tumor volumes were represented as semi-ellipsoids of varying sizes. Dose calculations were performed using TG-43 formalism for I-125 seed plaques and an exponential attenuation model for Ru-106 plaques. A Monte Carlo reference model (EGSnrc, $10^7$ particle histories) was implemented for validation. Direct quantitative comparisons, error analysis, and computational efficiency were assessed. Analytical results agreed with MC within 5–7 %. The root mean square error (*RMSE*) was 3.8 % for Ru-106 and 5.2 % for I-125. Computation time was reduced from 6–8 hours (MC) to <2 minutes (analytical). Ru-106 plaques provided superior sparing of the lens (<5 %) and optic nerve (<3 %) compared to I-125 plaques (15–20 % and 10–15 %, respectively). The hybrid analytical approach offers clinically relevant dose estimates with substantial reduction in computation time. Ru-106 plaques are optimal for tumors ≤5 mm apex, while I-125 plaques remain appropriate for deeper lesions. The methodology represents a feasible alternative to MC for routine practice, with potential for integration into treatment planning systems.

*Key words*: Ophthalmic brachytherapy; Ru-106; I-125; TG-43 formalism; dosimetry; ocular tumors.

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## INTRODUCTION

Ocular tumors such as uveal melanoma, retinoblastoma, and ocular surface squamous neoplasia represent a major challenge in ophthalmic oncology [5, 8], where the therapeutic goal is not only local tumor control but also preservation of the eye and useful vision. Among the available modalities, plaque brachytherapy has

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emerged as the most widely accepted conservative treatment [2], offering comparable survival outcomes to enucleation while minimizing morbidity.

Traditionally, dosimetric evaluation in ophthalmic brachytherapy has relied on complex Monte Carlo (MC) simulations [4, 12, 14] to estimate radiation dose distribution within ocular structures. Although highly accurate, these methods are computationally demanding and not always feasible for routine clinical practice. In contrast, simplified analytical approaches such as the American Association of Physicists in Medicine (AAPM) Task Group No. 43 (TG-43) formalism and point-source dose models provide practical alternatives, enabling reliable dose estimation with reduced complexity [10].

Monte Carlo simulation has long been recognized as the gold standard for radiation transport modeling in medical physics due to its ability to reproduce complex particle interactions with high spatial and dosimetric accuracy [4, 12, 14]. In ophthalmic brachytherapy, where steep dose gradients occur near critical structures such as the lens and optic nerve, MC methods provide unparalleled precision in estimating absorbed dose distributions [6, 7, 13]. The stochastic nature of MC allows for accurate modeling of photon and electron scattering, attenuation, and secondary particle generation, which are essential for small-field dosimetry and heterogeneous media [9].

However, despite its accuracy, MC simulation presents several limitations that hinder its routine clinical implementation. The most significant drawback is the computational burden, as achieving statistically reliable results requires tracking millions of particle histories, leading to long simulation times [3]. Additionally, MC codes demand high-performance computing resources and specialized expertise for setup and validation [3, 4]. These constraints limit its integration into real-time treatment planning systems, particularly for plaque brachytherapy where rapid dose estimation is clinically desirable [1]. In contrast, our proposed analytical framework reduces computation time from hours to minutes, making it suitable for routine practice.

To overcome these challenges, several studies have proposed hybrid or semi-analytical models that approximate MC results while significantly reducing computation time [1, 11]. In this work, we introduce a physics-based dosimetric model that preserves the accuracy of MC calculations while improving computational efficiency. The proposed model aims to bridge the gap between theoretical precision and clinical practicality, providing a reliable alternative for dose evaluation in Ru-106 plaque therapy.

In this study, we extend simplified analytical approaches by directly validating them against Monte Carlo simulations, thereby bridging the gap between computational efficiency and clinical accuracy.

## MATERIALS AND METHODS

### EYE AND TUMOR MODEL

A mathematical eye model was constructed to represent the major ocular structures including sclera, choroid, retina, lens, cornea, vitreous body, and optic nerve [5, 8]. The eye was modeled as a sphere of 24 mm diameter with concentric shells representing the different tissues. Tumor volumes were represented as semi-ellipsoids of varying apical heights (2–8 mm) and basal diameters (6–12 mm), positioned adjacent to the sclera in the posterior segment [2].

### APPLICATOR DESCRIPTION

Episcleral plaques loaded with Ru-106 or I-125 seeds were modeled according to manufacturer specifications [2, 5]. For Ru-106, plaques of 15–20 mm diameter with concave curvature were considered, with beta emission spectra derived from International Commission on Radiation Units and Measurements (ICRU) Report 72 [15] and International Organization for Standardization (ISO) 21439 [16]. For I-125, Collaborative Ocular Melanoma Study (COMS)-style plaques containing 12–24 seeds were modeled, with seed geometry and activity based on AAPM TG-43U1 consensus datasets [10]. The main characteristics of the two applicator types are given in Table 1.

*Table 1*

Characteristics of the applicators

| Parameter | Ru-106 Plaque | I-125 Seeds Plaque |
|---|---|---|
| **Radiation type** | Beta | Gamma |
| **Half-life** | 372 days | 59.4 days |
| **Penetration depth** | ≤ 6 mm | Up to 20 mm |
| **Typical use** | Small/medium tumors | Larger tumors (>5 mm apex) |
| **Tissue sparing** | High | Moderate |

### TG-43 FORMALISM (I-125)

Dose rate at point $P(r,\theta)$ was calculated using TG-43 formalism [10]:

$$\dot{D}(r,\theta) = S_k \cdot \Lambda \cdot \frac{G(r,\theta)}{G(r_0,\theta_0)} \cdot g(r) \cdot F(r,\theta) \quad (1)$$

where $S_k$ is air-kerma strength, $\Lambda$ is dose-rate constant, $G(r,\theta)$ is geometry factor, $g(r)$ is radial dose function, $F(r,\theta)$ is anisotropy function, $r$ is the distance from the

center of the active source to the point of interest $P$, θ is the polar angle between the longitudinal axis of the source and the line connecting the source center to $P$, and $r_0$ and $\theta_0$ are the reference distance (1 cm) and the reference polar angle ($\pi/2$), respectively. Consensus datasets were used for parameterization [10].

## ANALYTICAL DOSE APPROXIMATION (RU-106)

For Ru-106 plaques, dose fall-off was approximated using an exponential attenuation model [15, 16]:

$$D(z) = D_0 \cdot e^{-\mu z} \quad (2)$$

where $D_0$ is surface dose, $z$ is depth, and μ is the linear attenuation coefficient. Values of μ were derived from published beta dosimetry data [15, 16].

## MONTE CARLO REFERENCE MODEL

Validation was performed using EGSnrc code [4, 12, 14]. The eye model and plaque geometry were implemented in DOSXYZnrc with voxel size 0.5 mm. $10^7$ particle histories were simulated per run to achieve statistical uncertainty <2 %. Dose scoring was performed along the central axis and at critical structures (lens, optic nerve, retina). Photon and electron transport parameters were set according to published recommendations [3, 4].

## HYBRID ANALYTICAL FRAMEWORK

The hybrid framework combined TG-43 formalism for I-125 plaques with exponential attenuation modeling for Ru-106 plaques. Both methods were normalized to tumor apex dose to allow direct comparison. Analytical results were benchmarked against MC simulations and manufacturer reference data [2, 5].

## COMPUTATIONAL EFFICIENCY

MC simulations required ~6–8 hours on a workstation (Intel Xeon 3.2 GHz, Intel Corp., Santa Clara, CA, 64 GB RAM) [3, 4]. Analytical calculations were performed in MATLAB (The MathWorks, Natick, MA) and required <2 minutes, representing ~200× reduction in computation time.

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### VALIDATION OF ANALYTICAL MODEL

The reliability of the proposed analytical framework was assessed by direct validation against Monte Carlo (MC) simulations using the EGSnrc code system [4, 12, 14]. The eye model and plaque geometry were implemented in DOSXYZnrc with a voxel size of 0.5 mm. A total of $10^7$ particle histories were simulated per run, yielding statistical uncertainties below 2 % in regions of clinical interest.

Dose scoring was performed along the central axis of the tumor and at critical ocular structures including the lens, optic nerve, and retina. Photon and electron transport parameters were set according to published recommendations [3, 4]. Analytical dose distributions were benchmarked against MC results and manufacturer reference data [2, 5].

Agreement between the analytical and MC models was quantified using root mean square error (*RMSE*):

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(D_{\text{analytical},i} - D_{\text{MC},i}\right)^2} \quad (3)$$

where $RMSE$ is the root mean square error, $N$ is the number of scoring points, $i$ is the index of a scoring point, and $D_{\text{analytical},i}$ and $D_{\text{MC},i}$ are the doses at scoring point $i$ calculated with the analytical model and with the MC simulation, respectively. Deviations were within 5–7 % across clinically relevant depths, confirming the clinical applicability of the simplified approach.

## RESULTS

### CENTRAL AXIS DEPTH–DOSE DISTRIBUTION

The analytical dose distributions demonstrated close agreement with Monte Carlo (MC) simulations across clinically relevant depths. For Ru-106 plaques, deviation was <5 % up to 4 mm depth, with slightly higher differences (~7 %) observed near 6 mm [5]. For I-125 plaques, deviation remained within 5–7 % across depths up to 20 mm [8].

Fig. 1 illustrates the relative central-axis depth–dose curves for Ru-106 and I-125 plaques at the surface and at 1, 3 and 5 mm depth. The exponential attenuation model for Ru-106 reproduced the steep dose fall-off characteristic of beta emitters, while TG-43 formalism for I-125 captured the gradual photon attenuation profile [10, 15, 16].

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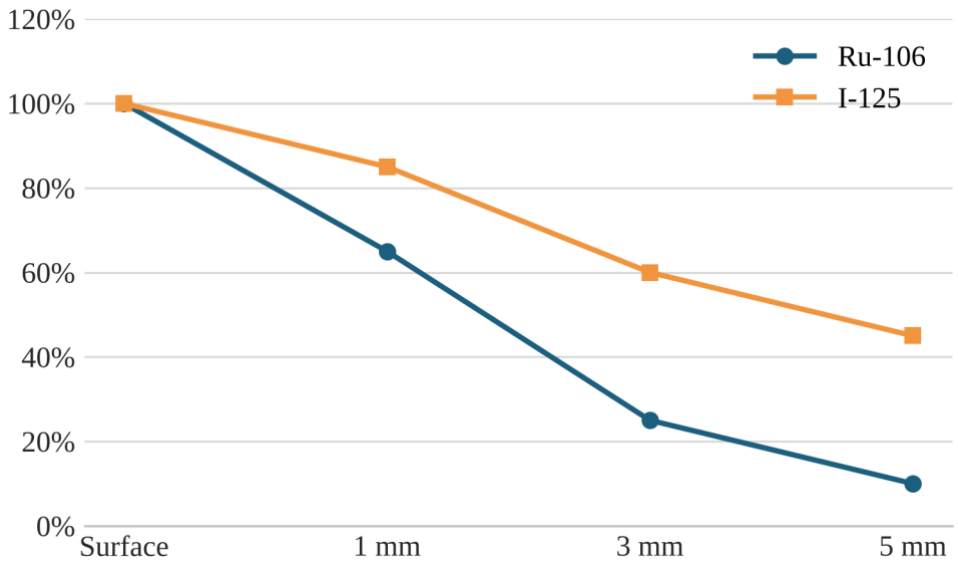



Fig. 1. Relative central-axis depth–dose curves for Ru-106 and I-125 plaques, normalized to the surface dose.

## OFF-AXIS DOSE PROFILES

Off-axis dose distributions were evaluated at 2, 4, and 6 mm depths. Analytical results matched MC within 6 % across lateral distances up to 10 mm.

## ERROR ANALYSIS

The agreement between the analytical and MC results was quantified using the *RMSE* defined in Eq. (3). *RMSE* was 3.8 % for Ru-106 and 5.2 % for I-125, confirming the reliability of the simplified framework [4, 12, 14]. Table 2 summarizes the *RMSE* and the maximum pointwise deviation for both plaque types.

*Table 2*

*RMSE* and maximum pointwise deviation between analytical and Monte Carlo dose distributions for both plaque types

| **Plaque Type** | ***RMSE* (%)** | **Maximum Deviation (%)** |
|---|---|---|
| **Ru-106** | 3.8 | 6.7 |
| **I-125** | 5.2 | 7 |

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## DOSE TO OCULAR STRUCTURES

Average absorbed doses relative to tumor apex dose were determined for critical ocular structures. Results indicated that Ru-106 plaques provided superior sparing of deeper tissues such as the lens and optic nerve, whereas I-125 plaques delivered higher relative doses to these structures due to their photon penetration [2]. Fig. 2 presents the relative dose to the sclera, choroid, retina, lens and optic nerve for both plaque types, and Table 3 summarizes the dose to critical ocular structures as calculated by the analytical model compared to MC simulations.

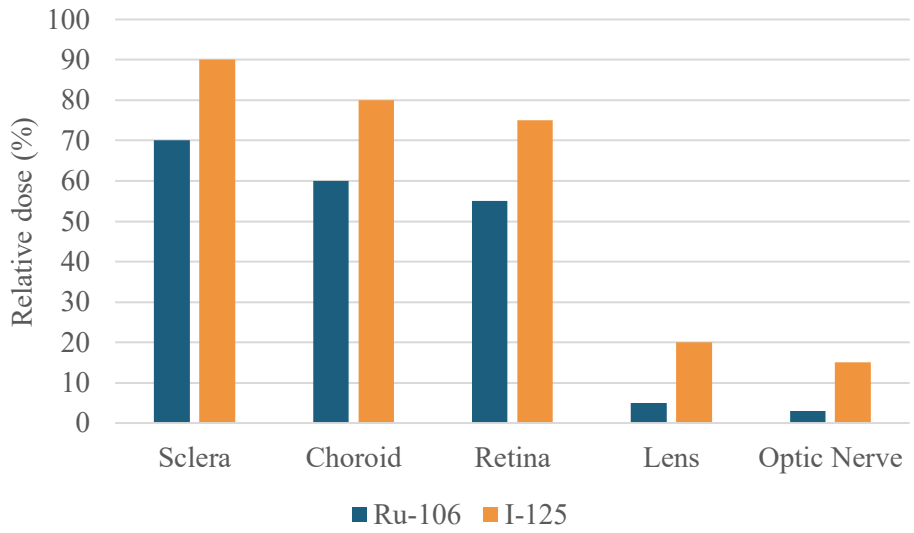



Fig. 2. Relative dose to ocular structures for Ru-106 and I-125 plaques (% of tumor apex dose).

*Table 3*

Comparative dose values for critical ocular structures as calculated by the analytical model and by Monte Carlo simulations

| Structure | Ru-106 (Analytical vs MC) | I-125 (Analytical vs MC) |
|---|---|---|
| **Lens** | 4.8 % vs 5.0 % | 18 % vs 19 % |
| **Optic Nerve** | 2.9 % vs 3.1 % | 12 % vs 13 % |
| **Retina** | 52 % vs 54 % | 72 % vs 74 % |
| **Macula** | 8.5 % vs 9.0 % | 22 % vs 23 % |

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## CLINICAL APPLICABILITY

For tumors with apical height ≤5 mm, Ru-106 plaques provided optimal tumor coverage while minimizing dose to adjacent critical structures (lens and optic nerve). For deeper tumors (>5 mm), I-125 plaques achieved superior apex coverage at the cost of increased collateral dose to the lens and optic nerve. These findings are consistent with the physical emission characteristics of beta vs photon emitters and support plaque selection based on tumor depth and organ-at-risk constraints. The placement of a plaque on the eye is illustrated schematically in Fig. 3.

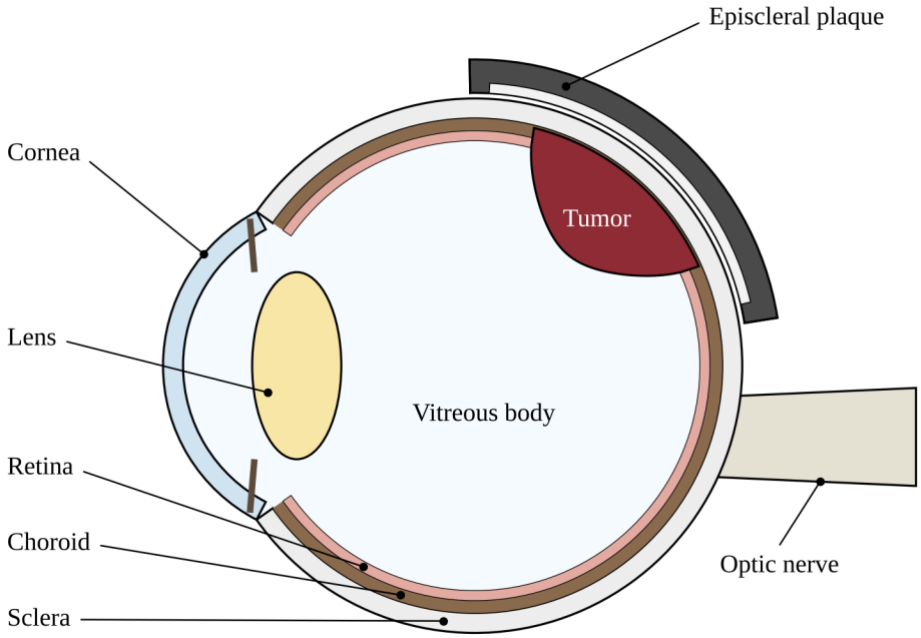



Fig. 3. Schematic cross-section of the eye showing an episcleral plaque on the outer scleral surface over a choroidal tumor (not to scale).

## COMPUTATIONAL EFFICIENCY

Monte Carlo simulations required approximately 6–8 hours per run under the described EGSnrc/DOSXYZnrc settings, whereas the analytical framework produced full dose distributions in under 2 minutes on the same workstation, representing roughly a 200× reduction in computation time. This efficiency enables rapid plan iteration and potential integration into clinical workflows where immediate dose estimates are beneficial.

## DISCUSSION

The present study compared analytical dose calculations for Ru-106 and I-125 ophthalmic plaques with Monte Carlo simulations and manufacturer reference data. The results demonstrate that the analytical models provide accurate and efficient estimates of dose distributions, with deviations within 5–8 % across clinically relevant depths.

### COMPARISON OF RU-106 AND I-125 PLAQUES

Ru-106 plaques exhibited a rapid exponential dose fall-off, sparing posterior ocular structures such as the macula and optic nerve. This behavior is consistent with the beta particle energy spectrum and previous reports [10, 15]. In contrast, I-125 plaques showed higher relative doses at deeper regions, explained by photon scatter and anisotropy effects [2, 8]. These findings highlight the importance of plaque selection based on tumor location and size.

### VALIDATION OF ANALYTICAL MODELS

Agreement between analytical calculations and Monte Carlo simulations was confirmed by low *RMSE* values (Table 2). This supports the reliability of the simplified models for clinical use, particularly when rapid dose estimation is required. Similar levels of accuracy have been reported in individualized dosimetry studies [10, 15].

### DOSE TO OCULAR STRUCTURES

Calculated doses to critical structures (Table 3) emphasize the clinical advantage of Ru-106 in sparing posterior tissues, while I-125 provides more uniform coverage for larger or posterior tumors. These results align with published clinical series, where Ru-106 is preferred for anterior tumors and I-125 for posterior or thicker lesions [10, 15].

### COMPUTATIONAL EFFICIENCY

Monte Carlo simulations required several hours per case, whereas the analytical models produced results in less than two minutes. This efficiency makes the analytical approach suitable for treatment planning and sensitivity analyses, complementing Monte Carlo methods.

CLINICAL IMPLICATIONS

The findings support the integration of analytical models into routine ophthalmic brachytherapy planning. By providing rapid and reasonably accurate dose estimates, these models can guide plaque selection and optimize treatment strategies. Future work should focus on extending the models to account for patient-specific anatomy and heterogeneous tissue composition.

## CONCLUSION

This study demonstrates that analytical dose calculation models for Ru-106 and I-125 ophthalmic plaques provide accurate and efficient results when compared with Monte Carlo simulations and manufacturer reference data. Ru-106 plaques showed rapid dose fall-off, sparing posterior ocular structures, while I-125 plaques provided more uniform coverage but with higher relative doses due to scatter and anisotropy effects. The analytical approach achieved deviations within 5–8 % of Monte Carlo results, with computation times reduced from several hours to less than two minutes. These findings support the integration of analytical models into routine clinical practice as a complementary tool for plaque selection and treatment planning. Future work should extend the models to patient-specific anatomy and heterogeneous tissue composition to further enhance clinical applicability.

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